NON-PARAMETRIC TESTS

Dr. Ahammad Basha Shaik, M.Sc., M.Phil., Ph.D.,
Assistant Professor (Statistics),
DEPARTMENT OF COMMUNITY MEDICINE,
NARAYANA MEDICAL COLLEGE,
Nellore - 524 003, Andhra Pradesh, India.

OUTLINE :-

- **Difference between parametric & non-parametric tests,**
- **Advantages and Disadvantages of Non-Parametric tests,**
- **When to use Parametric and Non-Parametric tests..,**
- **Types of non-parametric tests,**
    - **Chi-square test**
    - **Spearman's rank correlation coefficient**
    - **Sign test**
    - **Wilcoxon-Signed rank test**
    - **Mann-Whitney U test,**
    - **Kruskal-Wallis test.**
- **Practical exercises for the above tests using Microsoft Excel.**

Parametric and Non-Parametric tests

Parametric Tests :-

- If the information about the population is completely known by means of its parameters then statistical test is called parametric test.
- Parametric tests are restricted to data that:
- 1. show a normal distribution
- 2. are independent of one another
- 3. are on the same continuous scale of measurement

Non-Parametric Tests :-

- If there is no information about the population but still it is required to test the hypothesis of the population, then statistical test is called non-parametric tests.
- Non-Parametric tests are restricted to data that:
- 1. show an other-than normal distribution
- 2. are dependent or conditional on one another
- 3. in general, do not have a continuous scale of measurement
- For example, The height and weight of something -> **Parametric** ***Vs*** Did the bacteria grow or not -> **Non-Parametric**

Parametric and Non-Parametric tests

Parametric Tests :-

- Parametric tests are normally involve to data expressed in absolute numbers or values rather than ranks; an example is the Student's t-test.
- The results of a parametric test depends on the validity of the assumption.
- Parametric tests are most powerful for testing the significance.

Non-Parametric Tests :-

- Where we can not use the assumptions & conditions of parametric statistical procedures, in such situation we apply non-parametric tests.
- It covers the data techniques that do not rely on data belonging to any particular distribution.
- In this statistics is based on the ranks of observations and do not depend on any distribution of the population,
- In non-parametric statistics, the techniques do not assume that the structure of a model is fixed.
- It deals with small sample sizes, and, these are user friendly compared with parametric statistics and economical in time.

Parametric and Non-Parametric tests

Parametric tests	Non-parametric tests
It makes assumptions about the parameters of the population distribution(s) from which one's data are drawn	It makes no such assumptions
The information about the population is completely known by means of its parameters	There is no information about the population but still it is required to test the hypothesis of the population,
The data should be normally distributed	Data does not follow any specific distribution, So, it is known as ***"distribution free tests"***
Null hypothesis is made on parameters of the population distribution	The null hypothesis is free from parameters
It is applicable only for variable	It is applicable for both variables and attributes
No parametric tests for Nominal scale data	It exists for nominal and ordinal scale data
It is most powerful	It is not as powerful as parametric test

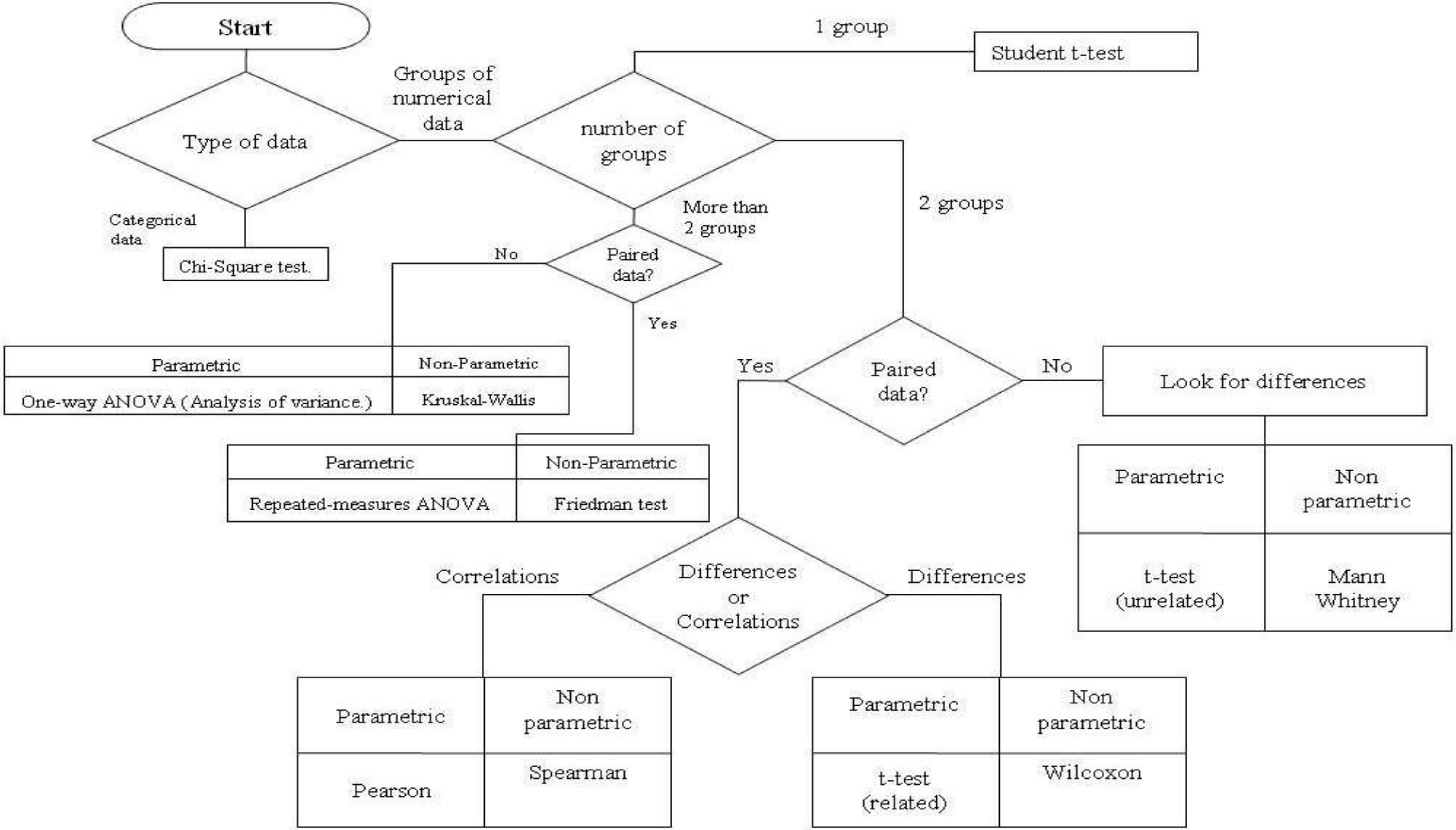
NON-PARAMETRIC TESTS

A precise and universally acceptable definition of the term "nonparametric" is not presently available – John E. Walsh

(Source: Handbook of Nonparametric Statistics, Volume 1, Chapter 1, p. 2)

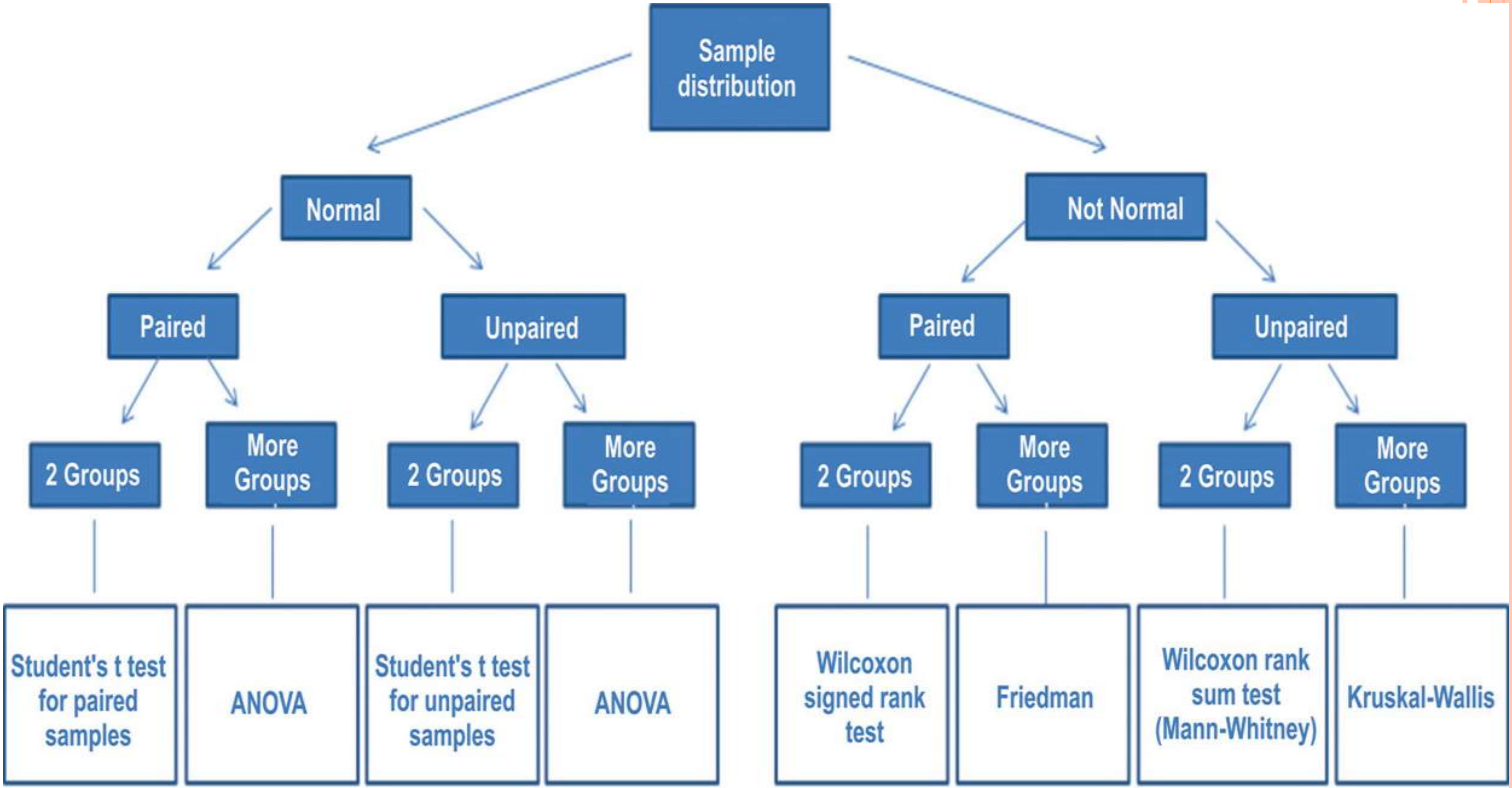
Advantages and Disadvantages of Non-Parametric tests

Advantages	Disadvantages
Non-parametric tests are simple and easy to understand	For any problem, if any parametric test exist it is highly powerful
It will not involve complicated sampling theory	Non-parametric methods are not so efficient as of parametric test
No assumption is made regarding the parent population	No nonparametric test available for testing the interaction in ANOVA model
Non-parametric test basically just need nominal or ordinal data	Tables necessary to implement non-parametric tests are scattered widely and appear in different formats
It is easy to applicable for attribute dates	May waste information
Non-parametric statistics are more versatile tests	Require a larger sample size than corresponding parametric test in order to achieve same power
Easier to calculate	Difficult to compute by hand for large samples
The hypothesis tested by the non-parametric test may be more appropriate	Stat tables are not readily available



Start
Type of data
Groups of numerical data
Categorical data
Chi-Square test.
number of groups
1 group
Student t-test
2 groups
More than 2 groups
Paired data?
No
Yes
Parametric
Non-Parametric
One-way ANOVA (Analysis of variance.)
Kruskal-Wallis
Parametric
Non-Parametric
Repeated-measures ANOVA
Friedman test
Paired data?
Yes
No
Look for differences
Parametric
Non parametric
t-test (unrelated)
Mann Whitney
Differences or Correlations
Correlations
Differences
Parametric
Non parametric
Pearson
Spearman
Parametric
Non parametric
t-test (related)
Wilcoxon

Different types of statistical methods for Analyzing the data



STATISTICAL TESTS

(Non-Parametric Tests)

Chi-Square Test

- The entire large sample theory was based on the application of **“Normality test”**.
- Exact sampling distribution tests are.
  - T-test,
  - F-test, and
  - Chi-square test.
- In all the exact sample tests, the basic assumption is that *“ The population(s) from which sample(s) are drawn is (are) normal.”* i.e., Parent population is normally distributed.

Chi-Square Test

- The chi-square distribution is the most frequently employed statistical technique for the analysis of count (or) frequency data.
- The chi-square statistic is most appropriate for use with categorical variables. i.e., Qualitative data, such as marital status, whose values are the categories married, single, widowed, and divorced.
- The ***observed frequencies*** are the number of subjects or objects in our sample that fall into the various categories of the variable of interest.
- ***Expected frequencies*** are the number of subjects or objects in our sample that we would expect to observe if some null hypothesis about the variable is true.

Chi-Square Test

Applications of Chi-Square test:-

- Hypothesis testing procedures of chi-square test are:
  - Tests for proportions,
  - Tests of Association, and
  - Tests of Goodness-of-fit.
- It can also be used when more than two groups are to be compared.
- h x k contingency table. (h = No. of Rows, k = No.of Columns)

STEPS FOR CALCULATION OF CHI-SQUARE TEST PROCEDURE

(i) State the Null hypothesis:

H_0: Two drugs are Same, H_1: Two drugs are different

(ii) Calculate the expected frequencies and apply the chi-square test:

Expected Frequency = Row total x Column total / N

The test statistic for the chi-square test is

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

Where, O_i = Observed value, E_i = Expected value

(iii) Find the degree of freedom (d.f.).

d.f. = (c-1)(r-1)

Where c= No. of columns & r = No. of rows.

STEPS FOR CALCULATION OF CHI-SQUARE TEST PROCEDURE

(iv) Probability tables of chi-square test and draw the inferences accordingly.

(v) Draw the conclusions accordingly.

If

(i) $\chi^2_{\text{cal. value}} < \chi^2_{\text{tab. value}} \Rightarrow$ Accept the H_0 and Reject the H_1 (i.e., $P > 0.05$).

(ii) $\chi^2_{\text{cal. value}} > \chi^2_{\text{tab. value}} \Rightarrow$ Reject the H_0 and Accept the H_1 (i.e., $P < 0.05$).

CHI-SQUARE TEST (PROBLEM): -

1) In an experiment on the immunization of goats from anthrax the following results were obtained. Derive your inference on the efficacy of the vaccine.

	Died	Survived	Total
Inoculated with vaccine	2	10	**12**
Not Inoculated	6	6	**12**
Total	**8**	**16**	**24**

Answer:-

H0: Survival is associated with inoculation

Expected Frequency (E_i)= (Row total x Column total) / N

O_i	E_i	$(O_i - E_i)$	$(O_i - E_i)^2$
2	4	-2	4
10	8	2	4
6	4	2	4
6	8	2	4

$\chi^2 = 4/4 + 4/8 + 4/4 + 4/8 = 1 + 0.5 + 1 + 0.5 = 3$

The number of degrees of freedom: (r-1) (c-1) = (2-1) (2-1) = 1.

- The value of χ^2 at 5% level of significance for one (1) degree of freedom is 3.841.

Interpretation:-

- Hence there is no cause to suspect the hypothesis and the data do not suggest that the survival is associated with inoculation.

Spearman's rank correlation test

- The Spearman's rank correlation test is a measure of association based on the ordinal features of data.
- This method is simple to use and easy to apply.
- The Spearman's coefficient of rank correlation 'ρ' is given by,

$$\rho = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n^3 - n}$$

where,

'd' is the difference between the two ranks and 'n' is the number of ranks.

- The limits for rank correlation coefficient are $-1 \le \rho \le 1$.

Spearman's rank correlation (problem)

Science ranks	Social Ranks	Difference (d)	d^2
1	1	0	0
2	10	-8	64
3	5	-2	4
4	4	0	0
5	7	-2	4
6	2	4	16
7	6	1	1
8	11	-3	9
9	9	0	0
10	15	-5	25
11	14	-3	9
12	12	0	0
13	16	-3	9
14	13	1	1
15	3	12	144
16	8	8	64
		d^2	**350**

$p = 1- 6 \sum d^2 / (n2-n)$

corr (p)	**0.485294118**
n	16

SIGN TEST

- The sign test is particularly useful in situations in which quantitative measurement is impossible or inconvenient, but on the basis of superior or inferior performance it is possible to rank with respect to each other, the two members of each pair.
- The use of this test does not make any assumption about the form of the distribution of differences. The only assumption underlying this test is that the variable under investigation has a continuous distribution.

SIGN TEST

- The sign test can be of *two* types:

  (1) The one sample sign test,

  (2) The paired sample sign test.

In a ***one-sample sign test,*** we test the null hypothesis $\mu = \mu_0$ against an appropriate alternative on the basis of a random sample of size 'n', we replace each sample value greater than μ_0 with a plus sign and each sample value less than μ_0 with a minus sign and discard sample value exactly equal to zero.

In ***paired sample sign test***, the data relating to the collection of an accounts. In this problem, each pair of sample values can be replaced with a plus sign if the first value is greater than the second, a minus sign if the first value is smaller than the second, or be discarded if the two values are equal. Then we proceed in the same manner as in one-sample sign test.

SIGN TEST

- The sign test is the simplest of the non-parametric tests. It is based on direction (or signs for pulses or minuses) of a pair of observations and not on their numerical magnitude.

In *sign test*, we have to count,

- Number of +ve signs,
- Number of –ve signs,
- Number of 0's (i.e., which cannot be included either as positive or negative)

We take H_0: $p = 0.05$ (Null Hypothesis)

If S is the number of times the less frequent (-ve) sign occurs, then S has the binomial distribution with $p = \frac{1}{2}$.

SIGN TEST

- The critical value for a two-sided alternative at a = 0.05 can be conveniently found by the expression.

$$\mathbf{K = ((n-1)/2) – (0.98*\sqrt{n})}$$

- H_0 is *rejected*, if $S \leq K$ for the sign test.
- H_0 is *accepted*, if $S > K$ for the sign test.

SIGN TEST

- For large samples, ($n \geq 25$) for the sign test, the normal approximation to the binomial may be used, correcting for continuity. Since $p = 0.50$ for this, we have the mean equal to $1/2n$ and the standard deviation equal to $1/2\sqrt{n}$.
- The actual value of *z* can be computed using the formula.

$$Z = (X-np_0) / \sqrt{np_0 (1-p_0)}$$

Where X is the number of plus signs. The value obtained can then be compared to the critical value of Z which is appropriate for the direction of the test. A mentioned before, in the event of ties, all sign changes of 0 are dropped before evaluating the results.

If the smaller number of positive (+) and negative (-) signs is less than or equal (≤) to critical value then we reject H_0, other wise accept H_0.

SIGN TEST
(PROBLEM)

One-sample sign test

Marks	di	
66	6	
58	-2	-1
70	10	
60	0	
56	-4	-1
55	-5	-1
81	21	
76	16	
53	-7	-1
71	11	
66	6	
59	-1	-1
88	28	
73	13	
80	20	
	S = 5	

H0 : Md = 60, H1 : Md > 60

No.of + signs	9
No.of - signs	5
No.of 0's	1
Total	15
n	**14**

k = (n-1)/2 - (0.98*sqrt(n))

K = 2.833176

Interpretation: -

Since S > k, the null hypothesis is accepted.

Two-sample (paired) sign test

Before	After	difference (d)
38	41	-3
34	37	-3
45	44	1
28	27	1
27	33	-6
25	30	-5
41	38	3
36	36	0
30	32	-2
28	29	-1
34	33	1
35	32	3
40	37	3
42	43	-1
33	40	-7
		S = 6

No.of + signs	6
No.of - signs	8
No.of 0's	1
Total	15
n	**14**

k = (n-1)/2 - (0.98*sqrt(n))

K = 2.833175761

Interpretation:

Since S > k, the null hypothesis is accepted. There is no significant difference between before and after the treatment.

Wilcoxon signed-rank test

- The *Wilcoxon* signed-rank test is proposed by *Frank Wilcoxon* (1945) and it does not take into consideration the magnitude of the difference between two equal groups.
- It is more powerful than the sign test because it tests not only direction but also the magnitude of differences within pairs of matched groups.
- This test like the sign test deals with dependent groups made up of matched pairs of individuals and is not applicable to independent groups.

Wilcoxon signed-rank test (test procedure)

- The Wilcoxon signed-rank test applies in the case of a symmetric continuous distribution. Under this condition, we can test the null hypothesis (H_0: $\mu_1 = \mu_2$).
- Rank the differences of the paired observations without regard to sign and assign the ranks.
- Calculate the totals for number of +ve signs and –ve signs and take the consideration into the account for smallest totals of signed values. The critical value of *w* can be obtained from table.
- Test the null hypothesis, i.e., the null hypothesis (H_0) is rejected if the computed value of lowest signed value is less than or equal to (<=) the appropriate table value at the level of significance (say, 0.05).
- i.e., The criteria to accept (or) reject the null hypothesis is, if $W \leq$ critical value then reject H_0, otherwise Accept H_0.

Wilcoxon signed-rank test (test procedure)

- For large samples ($n \geq 15$), the sampling distribution of W_+ (or) W_- approaches to the normal distribution with mean (μ) and variance (σ^2), then
- The test statistic is,

$$Z = W_+ - \mu / \sigma$$

Where,

W_+ is the sum of positive ranks, and $\mu = \dfrac{n(n+1)}{4}$ $\quad \sigma^2 = \dfrac{n(n+1)(2n+1)}{24}$

Test the null hypothesis, i.e., the null hypothesis (H_0) is rejected if the computed value of lowest signed value is less than or equal to (<=) the appropriate table value at the level of significance (say, 0.05).

Wilcoxon Signed-Rank Test (Problem)

Wilcoxon Signed rank test

Before	After	Diff (d)	ABS (d)	Rank of ABS (d)	Positive Ranks	Negative Ranks
58.5	60.0	-1.5	1.5	2		2
60.3	54.9	5.4	5.4	7	7	
61.7	58.1	3.6	3.6	6	6	
69.0	62.1	6.9	6.9	10	10	
64.0	58.5	5.5	5.5	8	8	
62.6	59.9	2.7	2.7	4	4	
56.7	54.4	2.3	2.3	3	3	
63.6	60.2	3.4	3.4	5	5	
68.2	62.3	5.9	5.9	9	9	
59.4	58.7	0.7	0.7	1	1	
					53	**2**

H_0: There is no significance difference between before and after the treatment

n	10	
T cal. Value	2	**T Cal. Value is the minmum value of positive or negative ranks**
T crit.value	8	***table value for n=7 at two-tailed 5% los***

Interpretation:-

Since Tcal < T tab. Then we reject null hypothesis and there is a significant difference between the two groups

Mann-Whitney U test

- This non-parametric test for two samples was described by *Wilcoxon* and studied by *Mann* and *Whitney*.
- The Mann-Whitney U test is used to compare difference between two independent groups when the dependent variable is either ordinal or continuous, but not normally distributed.
- It is most widely used test as an alternative to independent sample t test (parametric).
- It works on *interval* and *ordinal* data.
- It can be used for very small samples.

MANN-WHITNEY U TEST *(TEST PROCEDURE)*

- The test statistic of Mann-Whitney U test is as follows:

$$\mathbf{U_1 = w_1 - n_1(n_1 + 1)/2}$$

$$\mathbf{U_2 = w_2 - n_2(n_2+1))/2}$$

Where,

w_1 and w_2 are the total ranks for group 1 and 2,

n_1 & n_2 are the independent ordered samples of size n_1 and n_2 respectively.

The test statistic for the value of U is the minimum of U_1 and U_2, and compare with critical values of U_1 and U_2 for level of significance (say, 0.05).

If the minimum value of U_1 or U_2 is less than or equal to table critical value, the null hypothesis is rejected (Significant) other wise it is accepted (Not significant) at the level of significance.

MANN-WHITNEY U TEST *(TEST PROCEDURE)*

- The test statistic of Mann-Whitney U test is as follows:

$$\mathbf{U = n_1 n_2 + (n_2(n_2+1))/2 - T}$$

Where,

n_1 & n_2 are the independent ordered samples of size n_1 and n_2 respectively,

T is the sum of ranks of the second group in the combined ordered sample.

Consequently, the null hypothesis (H_0) is rejected, if U $\leq$ critical value, other wise accept H_0.

- If *T* is significantly ***large*** or small then then null hypothesis is rejected.

$$\mathbf{Z = (U - n_1 n_2/2) / \sqrt{n_1 n_2 (n_1+n_2+1)/12}}$$ ***(for large sample, say, n_1 & n_2 > 20)***

- The Mann-Whitney U test is always superior to the t-test for decidedly non-normal populations.

Mann-Whitney U test (*problem*)

Mann-Whitney U test

A	B	Rank A	Rank B
63	75	1	5
69	88	4	10
78	64	6	2
91	82	11	9
80	93	8	12
	79		7
	67		3
5	**7**	**30**	**48**

$$U_1 = w_1 - n_1(n_1 + 1)/2$$

$$U_2 = w_2 - n_2(n_2+1))/2$$

$$U\ stat = \min(U_1, U_2)$$

H_0: Median(A) = Median (B)

w1	30
w2	48
U1	15
U2	20
U stat	**15**

Critical Region (5,7) at alpha 0.05 is 5 (table value)

Decision:-

Ustat > 5 — 15 > 5

Accept H0

KRUSKAL - WALLIS TEST

- This test was introduced by *W.H. Kruskal* and *W.A. Wallis* (1952), for testing the equality of means in the one-factor analysis of variance when avoid the assumption that the samples were selected from normal populations.
- The *Kruskal-Wallis* test is an alternative test to analysis of variance.
- The *Kruskal-Wallis* test is also called the *Kruskal-Wallis H test*, is a generalization of the rank sum test to the case of $k > 2$ samples.
- It is used to test the null hypothesis H_0 that k independent samples are from identical populations.

KRUSKAL - WALLIS TEST *(TEST PROCEDURE)*

- Let us suppose that n_i (i = 1,2,…., n) be the number of observations in the i^{th} sample.
- First, we combine all k samples and arrange the $n = n_1 + n_2 + \ldots\ldots + n_k$ observations in ascending order, substituting the appropriate rank from 1,2,….,n for each observation.
- The sum of the ranks corresponding to the n_i observations in the i^{th} sample is denoted by the random variable r_i.
- To test the null hypothesis H_0 that *k* independent samples are from identical populations, the test statistic of *Kruskal-Wallis* test is given by,

$$H = \frac{12}{n(n+1)} \sum_{j=1}^{k} \frac{R_j^2}{n_j} - 3(n+1)$$

If *h* falls in the critical region $H > \chi_\alpha^2$ with $\nu = k - 1$ degrees of freedom, reject H_0 at the level of significance; otherwise, accept H_0.

i.e., If $H \geq$ critical value then we reject H_0, otherwise we accept H_0.

KRUSKAL - WALLIS TEST *(PROBLEM)*

Operating time in hours for the 3 types of scientific pocket calculators before a recharge is required

			Ranks		
A	B	C	Rank (A)	Rank (B)	Rank (C)
4.9	5.5	6.4	4	8.5	15
6.1	5.4	6.8	12	7	18
4.3	6.2	5.6	1	13	10
4.6	5.8	6.5	2	11	16
5.3	5.5	6.3	6	8.5	14
	5.2	6.6		5	17
	4.8			3	
5	**7**	**6**	**25.0**	**56.0**	**90.0**
Total	***18***				

Here, n1 = 5, n2 = 7, n3 = 6, r1 = 25.0, r2 = 56.0, r3 = 90.0

H_0: The operating time in hours for the 3 types of scientific pocket calculators are same.

h = 12/(n(n+1))*(sum(ri^2/ni))-3(n+1)

h = 10.47

Critical region: chi-square at 5% los at 2 d.f. is 5.991

Interpretation :-

Since h = 10.47 fall in the critical region, i.e., h > 5.991, we reject the hypothesis that the operating times are different (Significant).


Thank You!